# Space-Time Kriging Surrogate Model to Consider Uncertainty of Time Interval of Torque Curve for Electric Power Steering Motor

Junyong Jang[1], Jin Min Lee[1], Su-gil Cho[2], Saekyeol Kim[1], Ji-Min Kim[1], Jung-Pyo Hong[1] and Tae Hee Lee[1]

[1]Department of Automotive Engineering, Hanyang University, 222 Wangsimni-ro, Seongdong-gu, Seoul 04763, Korea
[2]Technology Center for Offshore Plant Industries, Korea Research Institute Ships and Ocean Engineering, Daejeon 34103, Korea

**In rotating motors, the stator or the rotor slots are often assembled in a skew position to reduce the effect of the permeance harmonics caused by slots. However, an actual skew position in a manufactured motor is not matched precisely to the ideal skew position. Thus, the effect of the uncertainty of a skew position is investigated in terms of the vibrational characteristic. Then, a space-time kriging surrogate model is proposed to consider the uncertainty of a skew position in the design stage with the aid of a computer simulation. The proposed surrogate model handles a time variable having time interval and predicts values between time intervals. Then, the performance of the proposed model is verified with simulation data and reliability estimated by the proposed model is compared with real data measured from manufactured motors.**

***Index Terms*—Computer simulation, Design optimization, Metamodeling, Torque, Uncertainty**

## I. Introduction

Cogging torque of an electric power steering (EPS) motor influences significantly on the handling performance of a vehicle. The cogging torque is measured by the difference in the maximum torque and the minimum torque for 360 electrical degrees under no load conditions. Thus, the cogging torque is the representative value for every time interval. In conventional surrogate models, this representative value becomes an output variable to be predicted [1]. In earlier researches, to suppress the cogging torque of permanent magnet synchronous motor, skew of the stator or the rotor is often applied [2]. However, skew has uncertainty from the imprecise lamination of the stator or the rotor [3]. The uncertainty of skew significantly influences the cogging torque, and this effect becomes worse in a small motor. However, it is difficult to predict the cogging torque of the skewed rotor or stator using conventional surrogate models. This is because it is obtained by shifting the torque curve as much and as many as each skew angle and adding up the torque curve to be as much as the number of skew steps. In addition, the interval of the uncertainty becomes much smaller than the experimented time interval. Thus, a novel surrogate modeling technique is necessary to deal with the dense and entire torque curve with time variable. In this paper, a space-time kriging surrogate model (STKRG) is proposed to handle the time variable as well as the space variable. Then uncertainty of cogging torque is accurately predicted.

## II. Uncertainty of Time Interval of EPS Motor

In PMSM, when the skew is applied to the rotor, the electrical steel sheets of the rotor are stacked up continuously or in several steps by a certain angle. This angle is called as the skew angle. An electrical skew angle is expressed as follows:

$$\text{Electric skew angle} = pp \times \frac{360°}{m \times n} \tag{1}$$

where $pp$ is the number of pole pairs. $m$ is least common multiple (LCM) of the number of poles and slots and $n$ is the number of steps.

The target motor considered in this research is a small motor for an EPS with 6 poles and 9 slots. It applies 4 steps skew to reduce the cogging torque, as shown in Fig. 1. The electrical skew angle for each step is 15 degrees. In an industry that manufactures the target motor, the uncertainty of the skew angle is managed within 10% of each skew angle. Because of the 10% uncertainty of the skew angle, the cogging torque increases about 4.5 times. Comparison results caused by the uncertainty of the skew angle are shown in Fig. 2 and listed in Table I. The target motor is so small that 1.5 electric skew angle is only about 0.15 mm in terms of the arc length of the rotor core.

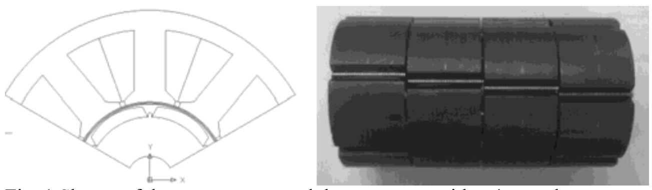

Fig. 1 Shapes of the target motor and the rotor core with a 4 step skew

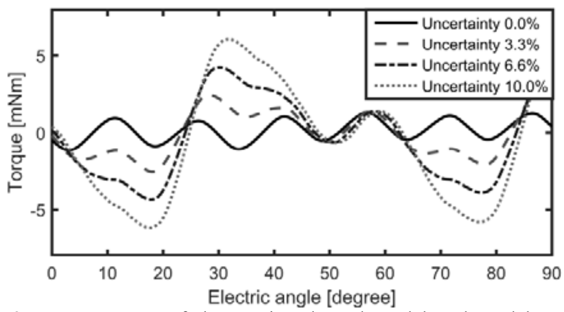


Fig. 2 Torque curves of the precise skewed model and models with the uncertainty of the skew angle

TABLE I
Cogging Torque According to the Uncertainty of the Skew Angle

| Electrical skew angle | Cogging torque [mNm] | Uncertainty |
|---|---|---|
| *Non-skewed* | 111.43 | N/A |
| *15.0 degree* | 2.86 | 0.0% |
| *15.5 degree* | 5.82 | 3.3% |
| *16.0 degree* | 9.35 | 6.6% |
| *16.5 degree* | 12.98 | 10.0% |

## III. SPACE-TIME KRIGING SURROGATE MODEL

Time interval of the simulation is determined by an analyst, and the cost increases when time interval is dense. From the torque value and time interval, the torque curve can be obtained. Thus, kriging surrogate models (KRGs) should be constructed as the number of time intervals. Multiple responses are $\mathbf{y}^1, \mathbf{y}^2, \cdots, \mathbf{y}^m$ at time interval $t^1, t^2, \cdots, t^m$ for space variables $\mathbf{x}^1, \mathbf{x}^2, \cdots, \mathbf{x}^n$. Then, KRGs by means of multiple responses becomes as follows:

$$\hat{\mathbf{y}}(\mathbf{x}, t)_{(1\times m)} = \mathbf{f}(\mathbf{x})^{\mathrm{T}}_{(1\times p)}\hat{\boldsymbol{\beta}}_{(p\times m)} + \ldots$$
$$\mathbf{r}(\mathbf{x})^{\mathrm{T}}_{(1\times n)}\mathbf{R}^{-1}_{(n\times n)}\left\{\mathbf{y}_{(n\times m)} - \mathbf{F}_{(n\times p)}\hat{\boldsymbol{\beta}}_{(p\times m)}\right\} \quad (2)$$
$$\hat{\boldsymbol{\beta}}_{(p\times m)} = \left\{\mathbf{F}^{\mathrm{T}}_{(p\times n)}\mathbf{R}^{-1}_{(n\times n)}\mathbf{F}_{(n\times p)}\right\}^{-1}\mathbf{F}^{\mathrm{T}}_{(p\times n)}\mathbf{R}^{-1}_{(n\times n)}\mathbf{y}_{(n\times m)}$$

where $p$ is the number of terms of the polynomial.

Equation (2) can predict multiple responses as many as the number of time intervals. However, the uncertainty of the skew angle is much smaller than the time intervals. Thus, it is necessary to predict the response between the time intervals. Thus, the STKRG is proposed to predict the responses at time untried points by means of the vector of the predicted responses $\hat{\mathbf{y}}$ and the presampled time points $t^1, t^2, \cdots, t^m$. Then, the STKRG for the responses with respect to the time variable becomes

$$\hat{\hat{\mathbf{y}}}(t)_{(1\times 1)} = \mathbf{f}(t)^{\mathrm{T}}_{(1\times p)}\hat{\hat{\boldsymbol{\beta}}}_{(p\times 1)} + \ldots$$
$$\hat{\mathbf{r}}(t)^{\mathrm{T}}_{(1\times m)}\hat{\mathbf{R}}^{-1}_{(m\times m)}\left\{\hat{\mathbf{y}}_{(m\times 1)} - \mathbf{F}_{(m\times p)}\hat{\hat{\boldsymbol{\beta}}}_{(p\times 1)}\right\} \quad (3)$$
$$\hat{\hat{\boldsymbol{\beta}}}_{(p\times 1)} = \left\{\mathbf{F}^{\mathrm{T}}_{(p\times m)}\hat{\mathbf{R}}^{-1}_{(m\times m)}\mathbf{F}_{(m\times p)}\right\}^{-1}\mathbf{F}^{\mathrm{T}}_{(p\times m)}\hat{\mathbf{R}}^{-1}_{(m\times m)}\hat{\mathbf{y}}_{(m\times 1)}$$

By using the STKRG, the responses at the arbitrary samples of time variable can be obtained and a smooth curve about the time variable can be achieved.

## IV. RESULTS AND CONCLUSION

The FE model analyzes 101 rotation steps. In terms of an electric angle, a time interval is 3.6 degrees. In this study, 163 samples are chosen to construct STKRG and two additional samples are selected for validation. Results are shown in Fig. 3 and listed in Table II. STKRG approximates low peak and high peak shapes in Fig. 3 (a), and a STKRG and a true curve is almost matched in Fig. 3 (b) by 1.49% error.

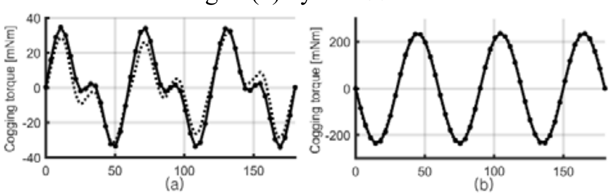


Fig. 3. Torque curves by FEA and curves predicted by STKRGs at additional 2 samples for validation; dot: validation samples, solid line: by FEA, dotted line: by STKRG

TABLE II
COMPARISON OF COGGING TORQUE

| No. | True (FEA) | STKRG | Relative error [%] | Unit |
|---|---|---|---|---|
| (a) | 68.66 | 63.39 | 7.67 | mNm |
| (b) | 473.75 | 466.68 | 1.49 | mNm |

The uncertainties could come from imprecise attachments of PMs and irregularly stacked sheets. Because permanent magnet (PM) is a piece, most of PM is close to a pre-determined skew angle. The uncertainty from PMs is assumed to be a normal distribution. On the other hand, most of the stacked sheets are out of alignment because dozens of sheets are randomly stacked. The uncertainty from the stacked sheets is assumed to be a bi-normal distribution. These two uncertainties occur independently, then the uncertainty of the skew angle can be assumed as combination of two normal distribution:

$$\text{Uncertainty} \sim \omega N\left(-0.6, 0.224^2\right) + (1-\omega)N\left(0.6, 0.224^2\right) \quad (4)$$

where $\omega$ is the weight factor, 0.5. A histogram of the uncertainty is shown in Fig. 4.

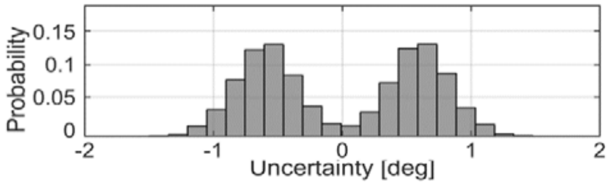


Fig. 4 Uncertainty of skew angle

The cogging torque of the 130 prototype motors are tested and it is compared with the Monte Carlo simulation (MCS) by using a STKRG. Comparison results are shown in Fig. 5 and listed in Table III. Fig. 5 (b) shows that the data are shifted to the left side compared with the real data, but the shape of the histogram is similar with real data in Fig. 5 (a). Thus, we can conclude that the ideal skewed models are rare in the real world like the assumption. Nevertheless, the reliabilities are very similar with real data. It is important to note that the result implies that the uncertainty of skew angle is well managed within ±10% error requirements in the industry.

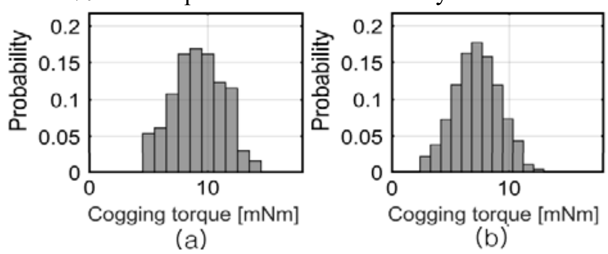


Fig. 5 Histograms of cogging torques; (a) real data, (b) MCS by STKRG

TABLE III
COMPARISON OF THE COGGING TORQUE AND RELIABILITY

| . | Minimum [mNm] | Maximum [mNm] | Reliability [%] = Pr(value < 15) |
|---|---|---|---|
| Real data | 5.00 | 14.00 | 100.00 |
| STKRG | 2.76 | 14.63 | 100.00 |

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